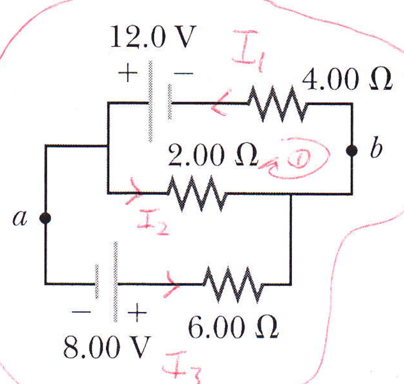
Name: Jake Bobowski Student Number: Sol'ns

Phys 102 Sec. 002 Assignment 3

Print this document and answer the questions in the space provided. Place a box around your final answer. Due Friday, April 1 @ 9:30 am.

1. **Kirchhoff's Rules... (5 marks)**

For the circuit given below, calculate (a) the current in each of the resistors and (b) the potential difference between points a and b.



(a) jcn rule:

$I_1 = I_2 + I_3$ ✱

loop rule:

① $-12V + 4I_1 + 2I_2 = 0$

$\therefore I_2 = 6 - 2I_1$

② $-12V + 4I_1 + 6I_3 - 8V = 0$

$\therefore I_3 = \frac{20}{6} - \frac{4}{6}I_1$

$= \frac{10}{3} - \frac{2}{3}I_1$

sub I_2 & I_3 into ✱

$I_1 = 6 - 2I_1 + \frac{10}{3} - \frac{2}{3}I_1$

$= \frac{18 - 6I_1 + 10 - 2I_1}{3}$

$= \frac{28}{3} - \frac{8}{3}I_1$

$\therefore 3I_1 = 28 - 8I_1$

$11 I_1 = 28 \Rightarrow I_1 = \frac{28}{11} = \boxed{2.55 A}$ (1)

$\therefore I_2 = 6 - 2I_1$

$= \boxed{0.900 A}$ (1)

$\therefore I_3 = \frac{10}{3} - \frac{2}{3}I_1$

$= \boxed{1.63 A}$ (1)

check:

$I_2 + I_3 = 2.53 A \approx I_1$ ✓

(b)

$V_b + I_2(2\Omega) = V_a$

$\therefore \Delta V = V_b - V_a$

$= -(2\Omega)I_2$

$= -1.80 V$ (1)

Va is at a higher potential.

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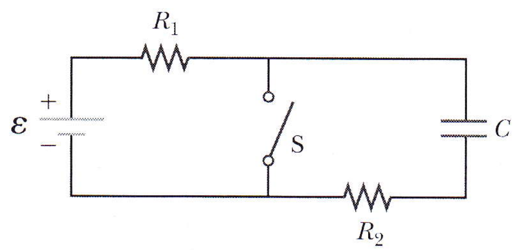
2. RC **Circuits. . . (4 marks)**

In the circuit shown in the figure below, the switch S has been open for a long time. It is then suddenly closed. Determine the time constant (a) before the switch is closed and (b) after the switch is closed. (c) Let the switch be closed at $t = 0$. Determine the current in the switch as a function of time.



(a) w/ switch open R_1 & R_2 are in series.

$\tau = RC$ where $R = R_1 + R_2$

$\therefore \boxed{\tau = (R_1 + R_2)C}$ (1)

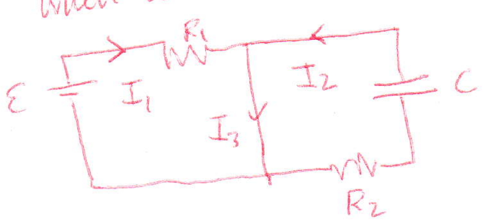
(b) w/ switch closed series combo of R_2 & C forms a closed loop.

$\therefore \boxed{\tau = R_2 C}$ (1)

(c) W/ switch initially open, capacitor charges to:

$C = \frac{Q_0}{\varepsilon} \Rightarrow Q_0 = C\varepsilon$

when switch is closed circuit becomes



jcn rule $I_1 + I_2 = I_3$ where I_3 is current in switch.

$\varepsilon - I_1 R_1 = 0 \Rightarrow I_1 = \varepsilon / R_1$

For discharging capacitor $q(t) = Q_0 e^{-t/\tau}$

$\therefore I(t) = \frac{dq(t)}{dt} = -\frac{Q_0}{\tau} e^{-t/\tau}$

Neg. sign means current dir'n opp. of dir'n when capacitor is charged.

$\therefore I_2 = \frac{Q_0}{R_2 C} e^{-t/R_2 C} = \frac{\varepsilon}{R_2} e^{-t/R_2 C}$

$\boxed{\therefore I_3 = \frac{\varepsilon}{R_1} + \frac{\varepsilon}{R_2} e^{-t/R_2 C}}$ (2)

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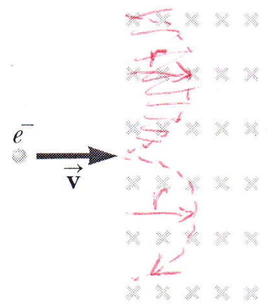
3. **Charge in Uniform $\vec{B}$... (4 marks)**

Assume the region to the right of a certain plane contains a uniform magnetic field of magnitude 1.00 mT and the field is zero in the region to the left of the plane as shown in the figure. An electron, originally traveling perpendicular to the plane, passes into the region of the field. (a) Determine the time interval required for the electron to leave the "field-filled" region. (b) Assuming that tge maximum depth of penetration into the field is 2.00 cm, find the kinetic energy of the electron.



Inside the field-filled region, the electron undergoes centripetal acceleration.

$$a_c = \frac{v^2}{r}$$ (1)

The magnetic force is

$$F_B = qvB = m\frac{v^2}{r}$$

$$\therefore v = \frac{qBr}{m}$$

$q = 1.60\times10^{-19}$ C
$B = 0.00100$ T
$r = 0.0200$ m
$m = 9.11\times10^{-31}$ kg

$$\therefore v = 3.51\times10^6 \text{ m/s}$$ (1)

angular speed is

$$\omega = \frac{v}{r} = \frac{\Delta\theta}{\Delta t}$$

$$\therefore \Delta t = \frac{\Delta\theta\, r}{v}$$

$\Delta\theta = \pi$ rad.

$$\boxed{\therefore \Delta t = 17.9 \text{ ns}}$$ (1)

K.E:

$$K = \frac{1}{2}mv^2$$

$$= \boxed{5.61\times10^{-18} \text{ J}}$$ (1)

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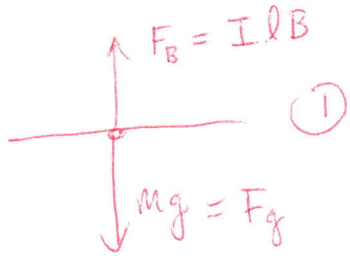
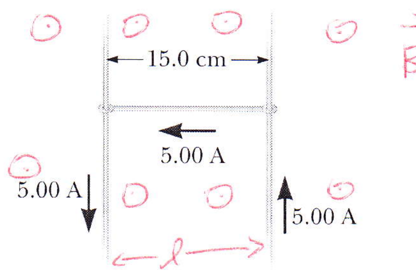
4. **Magnetic Force... (3 marks)**

Consider the scenario shown in the figure. A 15.0 cm horizontal wire of mass 15.0 g is placed between two thin, vertical conductors, and a uniform magnetic field acts perpendicular to the page. The wire is free to move vertically without friction on the two vertical conductors. When a 5.00 A current is directed as shown, the horizontal wire moves upward at constant velocity. Find the magnitude and direction of the magnetic field required to move the wire at constant speed.

Free body diagram for sliding wire



(1)



$\vec{F}_B = I\vec{\ell} \times \vec{B}$

In order that $\vec{F}_B$ directed upwards, must have $\vec{B}$ out of the page. (1)

In equilibrium $F_B = F_g$ & wire moves w/ const speed.

$$I\ell B = mg$$

$$\therefore B = \frac{mg}{I\ell}$$

$m = 0.0150\,kg$
$g = 9.80\,m/s^2$
$I = 5.00\,A$
$\ell = 0.150\,m$

(1)

$$\therefore B = 0.196\,T$$

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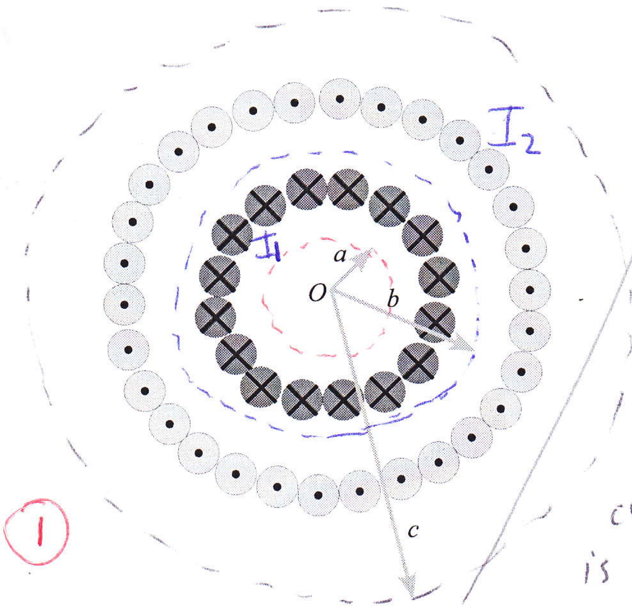
5. **Ampere's Law... (5 marks)**

In the figure below there N_1 long straight wires that each carry current I_1 into the page while another N_2 long straight wires each carry current I_2 out of the page. Use Ampere's Law to find the magnitude of the magnetic field a distance (a) a, (b) b, and (c) c from the origin O. (The wires are in a cylindrical arrangement. The two cylinders share a common axis that is coincident with the origin O.)



(a) Ampere's Law

$\oint \vec{B}\cdot d\vec{s} = \mu_0 I$

Pick amperian loop w/ radius a.

By symmetry $\vec{B} \parallel d\vec{s}$ & const. on loop.

$\oint \vec{B}\cdot d\vec{s} = \oint B\,ds = B\oint ds = B\,2\pi a$

For part (a) current through loop is zero.

$\therefore B(2\pi a) = 0 \Rightarrow \boxed{B=0}$ (1)

(1)

(b) Take the blue amperian loop.

Just as in (a) $\oint \vec{B}\cdot d\vec{s} = B(2\pi b)$

This time, however, current through the loop is $I = N_1 I_1$

$\therefore B(2\pi b) = \mu_0 N_1 I_1$

$$\boxed{B = \frac{\mu_0 N_1 I_1}{2\pi b}}$$ (1)

(c) take the black amperian loop. Here

$\oint \vec{B}\cdot d\vec{s} = B(2\pi c)$

current through this loop is $I = N_1 I_1 - N_2 I_2$

(currents are in opp. dir'ns) (1)

$\therefore B(2\pi c) = \mu_0 (N_1 I_1 - N_2 I_2)$

$$\boxed{\therefore B = \frac{\mu_0}{2\pi c}(N_1 I_1 - N_2 I_2)}$$

(1)

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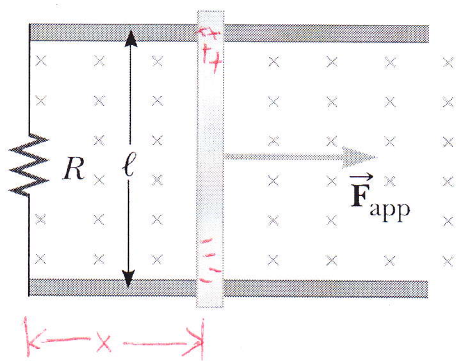
6. Motional EMF... (4 marks)

Consider the arrangement shown in the figure. Assume $R = 6.00\ \Omega$, $\ell = 1.20$ m, and a uniform 2.50 T magnetic field is directed into the page. At which speed should the bar be moved to produce a current of 0.500 A in the resistor?

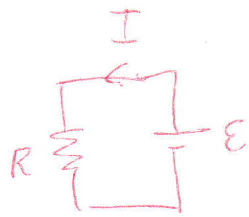


$\mathcal{E} = -\frac{d\Phi_B}{dt}$ (1)

$\Phi_B = AB$
$= \ell x B$

$\frac{d\Phi_B}{dt} = \ell B \frac{dx}{dt} = \ell B v = -\mathcal{E}$ (1)

Equivalent circuit is:



$I = \frac{\mathcal{E}}{R}$ (1)

$\therefore |I| = \frac{\ell B v}{R} \Rightarrow$ $\boxed{v = \frac{IR}{\ell B} = 1.00\ \text{m/s}}$ (1)

$R = 6.00\ \Omega$
$\ell = 1.20$ m
$B = 2.50$ T
$I = 0.500$ A

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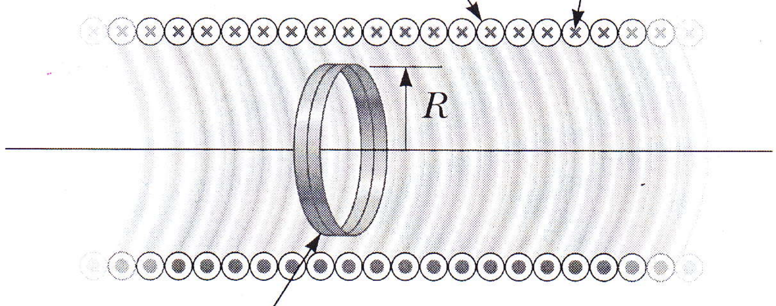
7. Induced emf... (5 marks)

A long solenoid has $n = 400$ turns per meter and carries a current given by $I = 30.0(1 - e^{-1.60t})$. Inside the solenoid and coaxial with it is a coil that has a radius of $R = 6.00$ cm and consists of a total of $N = 250$ turns of fine wire (see the figure below). What is the magnitude of the emf induced in the coil by the changing current?



Magnetic field inside solenoid is uniform. $B = \mu_0 n I$ (1)

Flux through ring inside solenoid is $\Phi_B = BA = \pi R^2 \mu_0 n I$ (1)

magnitude of induced emf is $|\mathcal{E}| = N\frac{d\Phi_B}{dt} = N\pi R^2 \mu_0 n \frac{dI}{dt}$ (1)

$\frac{dI}{dt} = (30.0)(1.60)e^{-1.60t}$ (1)

$N = 250$
$R = .0600$ m
$n = 400$ m^{-1}

$\therefore |\mathcal{E}| = 0.0682 e^{-1.60t}$ V $= \boxed{(68.2 e^{-1.60t})\text{ mV}}$ (1)

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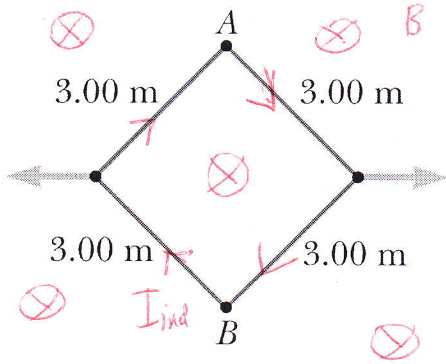
8. **Induced Current... (5 marks)**

The square loop in the figure is made of wires with a total series resistance of 10.0 Ω. It is placed in a uniform 0.100 T magnetic field directed perpendicular into the plane of the paper. The loop, which is hinged at each corner, is pulled as shown until the separation between the points A and B is 3.00 m. If this process takes 0.100 s, what is the average current generated in the loop? What is the direction of the current?



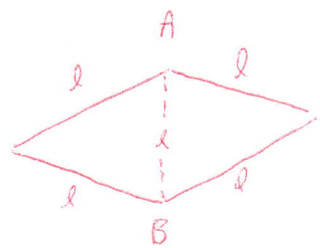
$\mathcal{E} = -\frac{d\Phi_B}{dt}$

$\mathcal{E} = -B\frac{\Delta A}{\Delta t}$ (1)

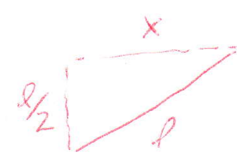
$\Delta A = A_f - A_i$

$A_i = \ell^2$ where $\ell = 3.00\,m$

Final shape:



$\frac{1}{4}$ of shape looks like:



$\frac{\ell^2}{4} + x^2 = \ell^2$

$\therefore x^2 = \frac{3\ell^2}{4}$

$x = \frac{\sqrt{3}}{2}\ell$

$A_f = 4\left(\frac{1}{2}\underbrace{\frac{\ell}{2}}_{base}\underbrace{\frac{\sqrt{3}}{2}\ell}_{height}\right) = \frac{\sqrt{3}}{2}\ell^2$ (1)

$\Delta A = A_f - A_i = \left(\frac{\sqrt{3}}{2} - 1\right)\ell^2 \Longrightarrow |\mathcal{E}| = \left|\frac{B\ell^2}{\Delta t}\left(\frac{\sqrt{3}}{2} - 1\right)\right| = \boxed{1.21\,V}$ (1)

$I = \frac{\mathcal{E}}{R} = \frac{1.21\,V}{10\,\Omega} = \boxed{0.121\,A}$ (1)

Area decreases ∴ flux decreases ∴ induced current generates $\vec{B}$ in same dir'n as applied $\vec{B}$.

Current is CW (1)